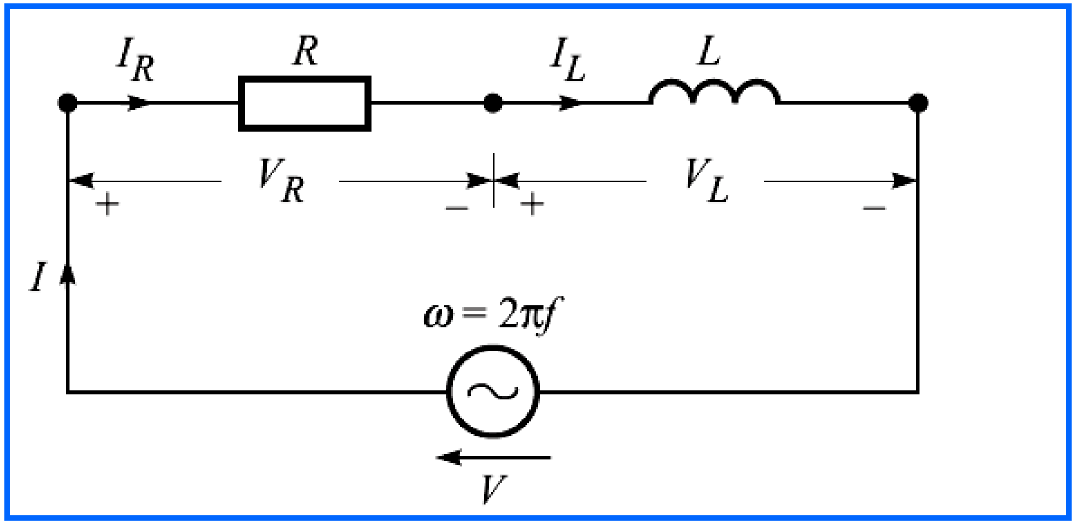
Topics to be Discussed

- **Series *RL* Circuit.**
- **How to Draw Phasor Diagram.**
- **Complex Impedance.**
- **Impedance Triangle.**
- **Series *RC* Circuit.**
- **Complex Power.**
- **Series *RLC* Circuit.**

Series *RL* Circuit



To write $V = V_R + V_L$ is **wrong**.

In AC circuits, the two voltages must be added by treating them as ***phasors***,

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L$$

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How to Draw Phasor Diagram

1. Mark the source voltage *V*, with its polarity. Mark the source current *I* leaving the positive terminal.
2. Mark 'the voltage across' and 'the current through' each individual component of the circuit, following the passive sign convention.
3. Draw the phasor diagrams for individual components.
4. Superimpose all the individual phasor diagrams, by recognizing the *common phasor* among them.
5. For this, you may have to rotate some phasor diagrams.
6. Find the phasor addition by drawing the parallelogram.

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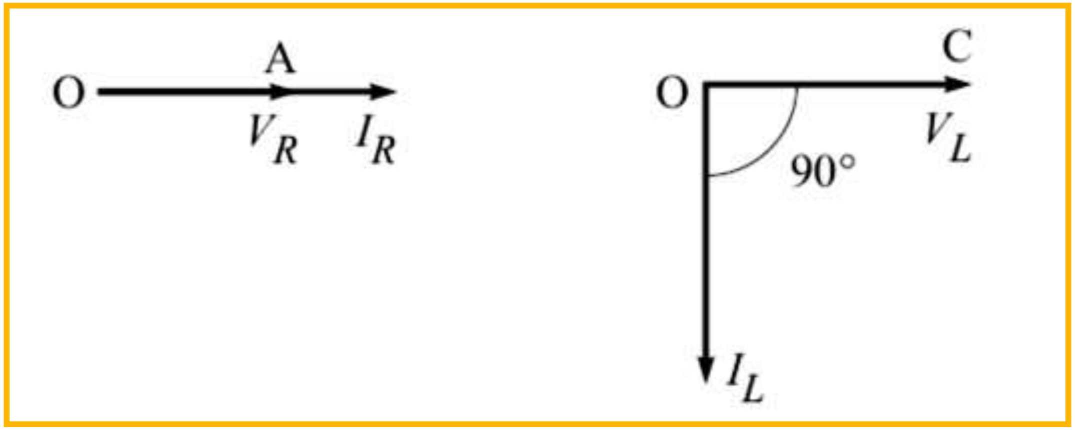
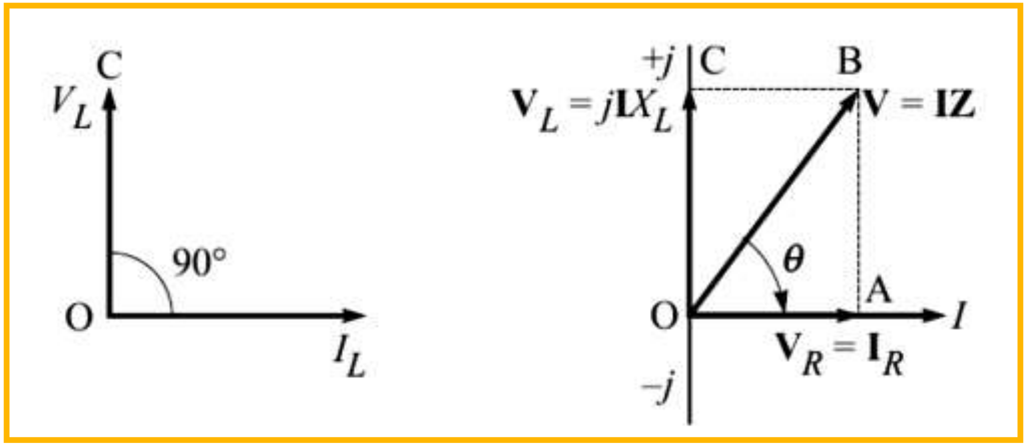


(b) Phasor diagram for R.

(c) Phasor diagram for L.



(d) Phasor diagram for L, rotated by +90°.

(e) The complete phasor diagram.

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Complex Impedance

Complex impedance

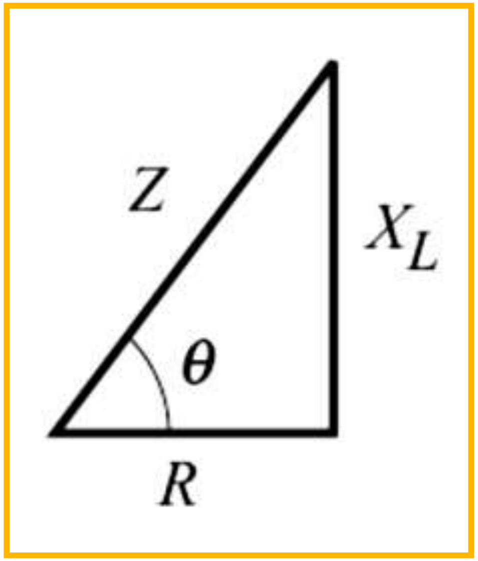
= (resistance) + j (reactance) or $\mathbf{Z}$ = $R + jX$

For the series *RL* circuit,

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R + j\omega L = Z\angle\theta$$

$$\text{where, } Z = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{\omega L}{R}$$

Impedance Triangle



- **We can separate the voltage triangle OAB.**
- **Dividing each side of this triangle by *I*, we get an impedance triangle.**
- **Note that an inductive circuit has an impedance triangle in the first quadrant of complex plane.**

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Often, we are given the source voltage.

We take voltage as reference and then find the resulting current.

Click

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{V\angle 0^\circ}{Z\angle\theta} = \frac{V}{Z}\angle-\theta$$

If $v = V_m \sin\omega t,$

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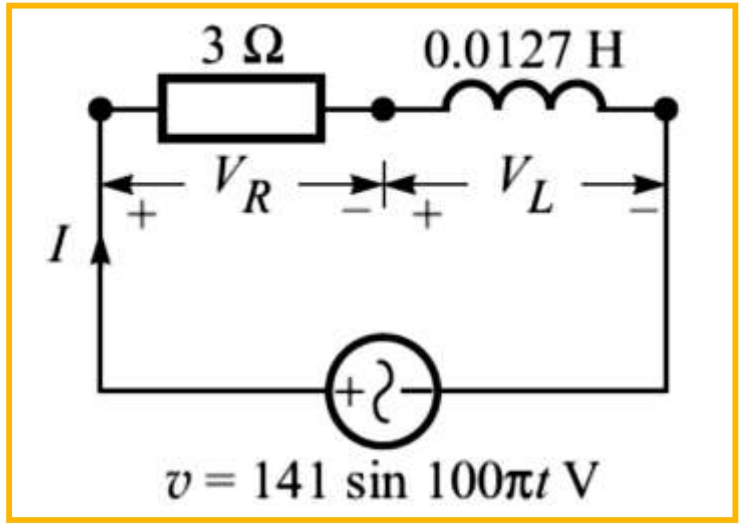
$$i = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \sin\{\omega t - \tan^{-1}(\omega L)/R\}$$

Example 1



For the series *RL* circuit shown,

Calculate the rms value of the steady state current and the relative phase angle.

Write the expression for the instantaneous current.

Find the average power dissipated in the circuit.

Determine the power factor.

Draw the phasor diagram.

Next

Solution : The applied voltage can be written as

$$\mathbf{V} = V\angle 0^\circ = \frac{V_m}{\sqrt{2}}\angle 0^\circ = \frac{141}{\sqrt{2}}\angle 0^\circ = 100\angle 0^\circ \text{ V}$$

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The impedance, $\mathbf{Z} = R + j\omega L = 3 + j100\pi \times 0.0127$

$$= 3 + j4 = 5\angle 53.1^\circ\,\Omega$$

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$$\therefore \quad \text{Current, } \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{100\angle 0^\circ}{5\angle 53.1^\circ} = \mathbf{20\angle -53.1^\circ\ A}$$

Click

$$\therefore \quad i = 20\sqrt{2}\sin(100\pi t - 53.1^\circ)$$

$$= \mathbf{28.28\sin(100\pi t - 53.1^\circ)\ A}$$

Next

$$P = VI\cos\theta = 100 \times 20 \times \cos 53.1° = \mathbf{1200\ W}$$

or $$P = I^2 R = (20)^2 3 = \mathbf{1200\ W}$$

$$pf = \cos\theta = \cos 53.1° = \mathbf{0.6\ lagging}$$

or $$pf = \frac{P}{VI} = \frac{1200}{100 \times 20} = \mathbf{0.6\ lagging}$$

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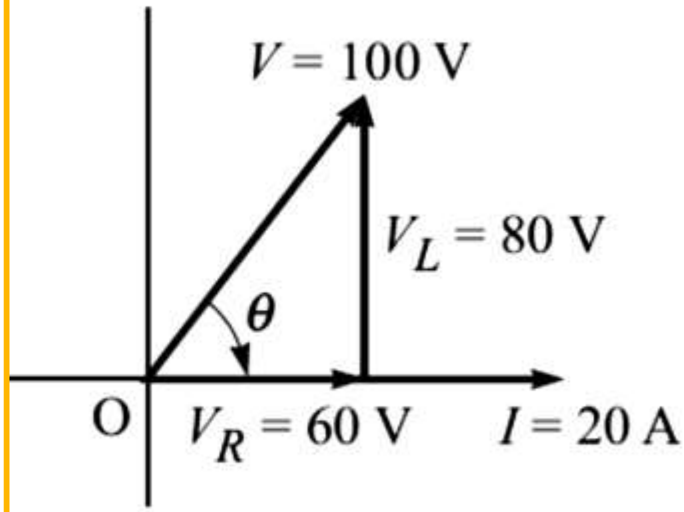
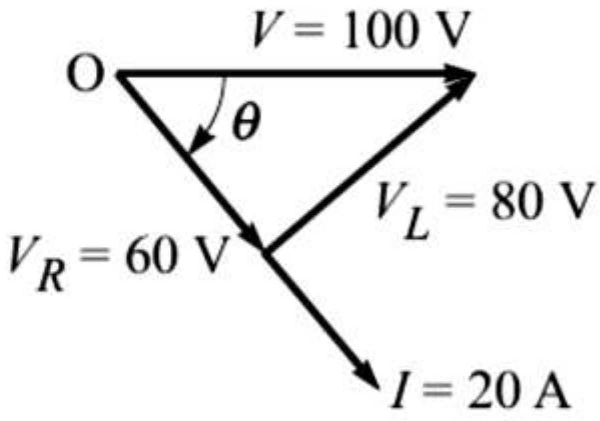


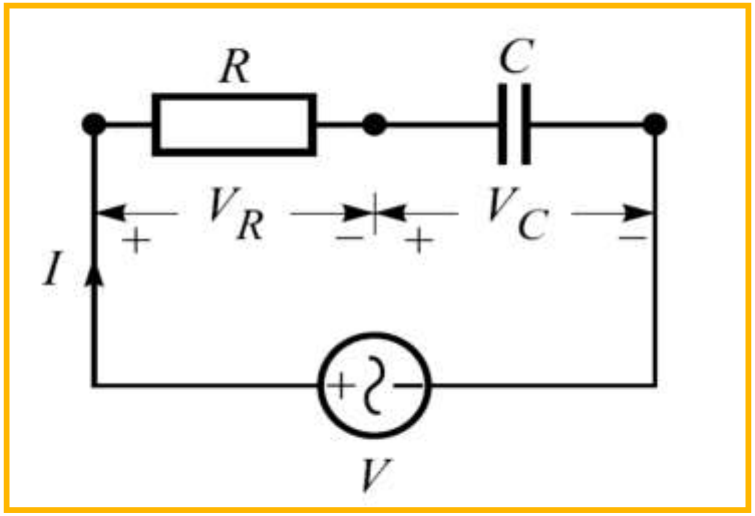
(*a*) Phasor diagram (with **I** as ref.)



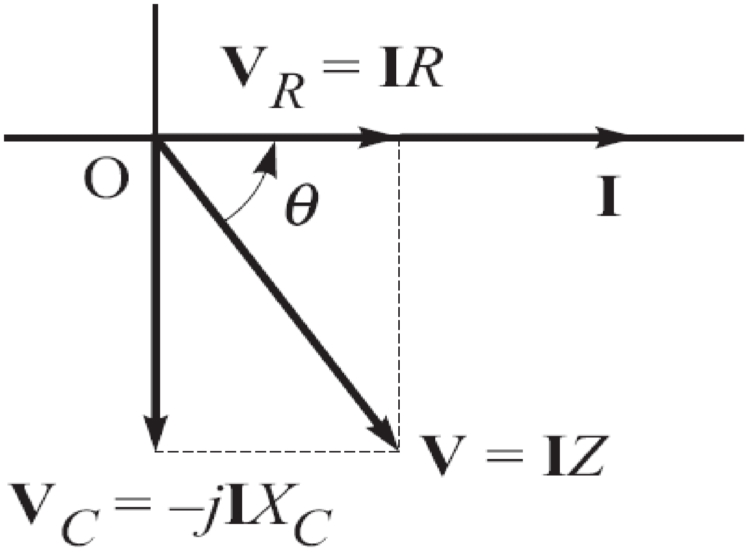
(*a*) Phasor diagram (with **V** as ref.)

Next

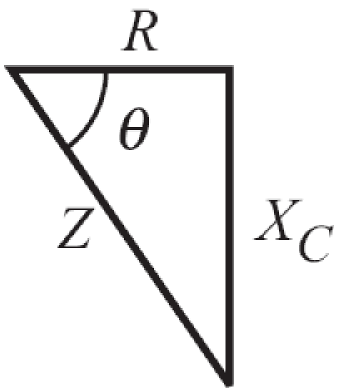
Series *RC* Circuit



(*a*) The circuit.



(*b*) Phasor diagram.



(*c*) Impedance triangle.

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By applying KVL, we get

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C = \mathbf{I}R - j\mathbf{I}X_C = \mathbf{I}(R - jX_C) = \mathbf{I}\left(R - j\frac{1}{\omega C}\right) = \mathbf{IZ}$$

Since the voltage across a capacitor lags the current by 90°.

- Note that $-j$ is associated with X_C

  (whereas $+j$ is associated with X_L).

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Complex Power

Let the terminal voltage, $\mathbf{V} = V\angle\theta$

and the current, $\mathbf{I} = I\angle\phi$

The average power absorbed, $P = VI\cos(\theta - \phi)$

By using Euler's formula, the above equation

$$P = VI\,\mathrm{Re}[e^{j(\theta-\phi)}] = \mathrm{Re}\,[(Ve^{j\theta})(Ie^{-j\phi})]$$

or $P = \mathrm{Re}[\mathbf{VI}^*]$

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We now define the ***complex power*** as

$$\mathbf{S} = P + jQ = \mathbf{VI}^* = VIe^{j(\theta-\phi)}$$

Note :

- **The magnitude *VI* of S is the apparent power (VA).**
- **The angle of S is the pf angle.**
- **The real part *P* of S is the real power (W).**

$$P = VI \cos(\theta - \varphi)$$

- **The imaginary part *Q* of S is reactive power (VAR),**

$$Q = VI \sin(\theta - \phi)$$

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Example 2

A current of 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω.

Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Solution : Let $\mathbf{I} = 0.9\angle 0^\circ$ A

Click

$$\mathbf{Z} = 120 - j250 = \mathbf{277.3\angle -64.4^\circ\ \Omega}$$

$$pf = \cos\theta = \cos(-64.4^\circ) = \mathbf{0.432\ leading}$$

Click

$$\mathbf{V} = \mathbf{IZ} = (0.9\angle 0^\circ)(277.3\angle -64.4^\circ) = \mathbf{249.6\angle -64.4^\circ\ V}$$

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$$\mathbf{V}_R = IR = (0.9\angle 0^\circ)\times 120 = \mathbf{108\angle 0^\circ\ V}$$

$$\mathbf{V}_C = IX_C = (0.9\angle 0^\circ)(250\angle -90^\circ) = \mathbf{225\angle -90^\circ\ V}$$

Click

$$P_{app} = VI = 249.6\times 0.9 = \mathbf{224.6\ VA}$$

$$P_a = VI\cos\theta = 249.6\times 0.9\times\cos 64.4^\circ = \mathbf{97.03\ W}$$

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$$P_r = VI\sin\theta = 249.6\times 0.9\times\sin 64.4^\circ = \mathbf{202.55\ VAR}$$

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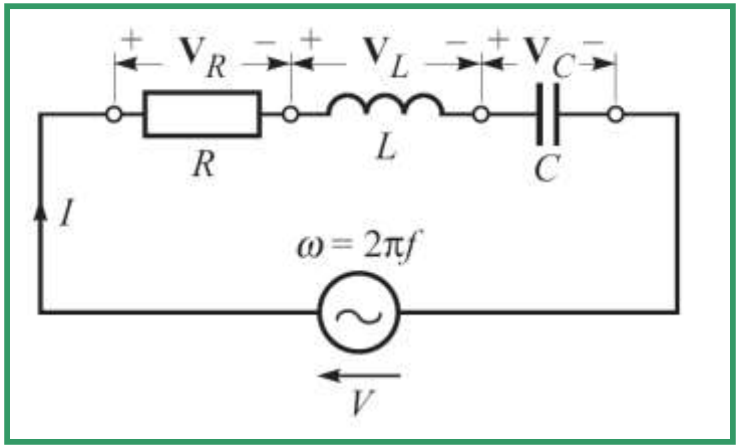
Next

Series *RLC* Circuit



(a) The circuit.

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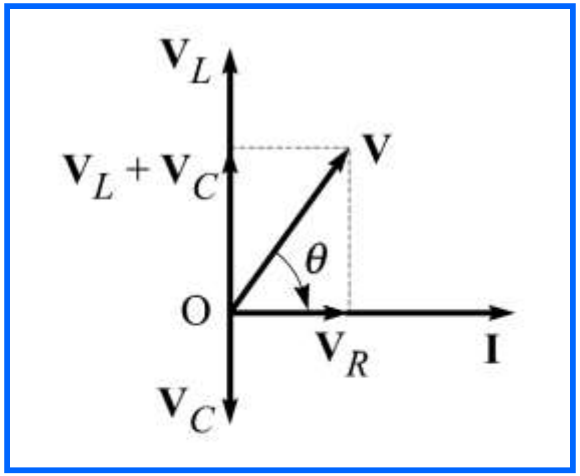
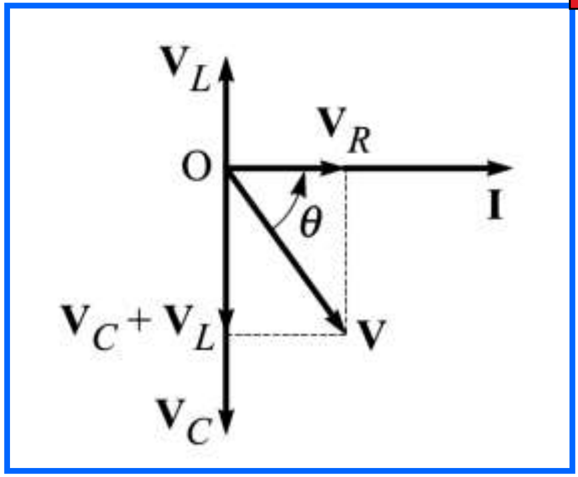


(b) Phasor diagram (when $\omega L > 1/\ \omega C$)



(c) Phasor diagram (when $\omega L < 1/\ \omega C$).

Next

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C = \mathbf{I}R + \mathbf{I}(jX_L) + \mathbf{I}(-jX_C)$$

$$= \mathbf{I}[R + j(X_L - X_C)] = \mathbf{IZ}$$

Click

$$\mathbf{Z} = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = |\mathbf{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

and $$\theta = \tan^{-1}\frac{\omega L - (1/\omega C)}{R}$$

Next

There can be following three possibilities :

(1) When ωL > 1/ ωC : The circuit behaves as an **inductive circuit.**

(2) **When ωL < 1/ ωC :** The circuit behaves as a **capacitive circuit.**

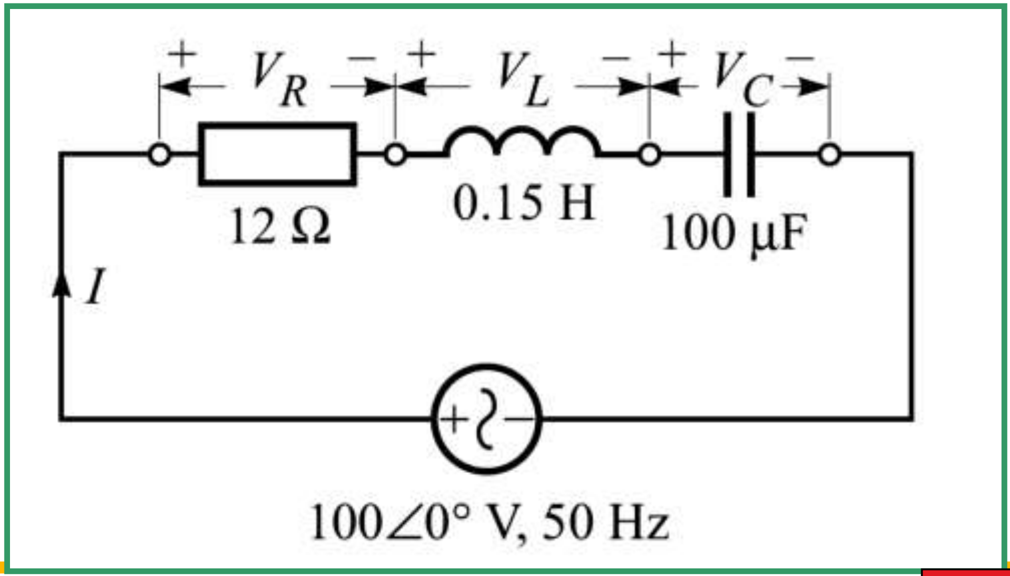
(3) When ωL = 1/ ωC : The circuit behaves as a ***purely resistive circuit***. This is a special case, and is called ***resonance***.

Next

Example 4

For the circuit shown, calculate (*a*) the impedance, (*b*) the current, (*c*) the phase angle, (*d*) the voltage across each element, (*e*) the power factor, (*f*) the apparent power, and (*g*) the average power.

Also, draw the phasor diagram.



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Solution :

$$X_L = \omega L = 2\pi f L = 100\pi \times 0.15 = 47.1\ \Omega;$$

$$X_C = 1/\omega C = 1/(100\pi \times 100 \times 10^{-6}) = 31.8\ \Omega$$

(a) The impedance,

$$\mathbf{Z} = R + j(X_L - X_C) = 12 + j(47.1 - 31.8) = \mathbf{(12 + j15.3)\ \Omega}$$

$$= \mathbf{19.4\angle 51.9^\circ\ \Omega}$$

(b) The current,

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{100\angle 0^\circ}{19.4\angle 51.9^\circ} = \mathbf{5.15\angle -51.9^\circ\ A}$$

(c) The phase angle,

$$\phi = -\mathbf{51.9^\circ}$$

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(e) The power factor,

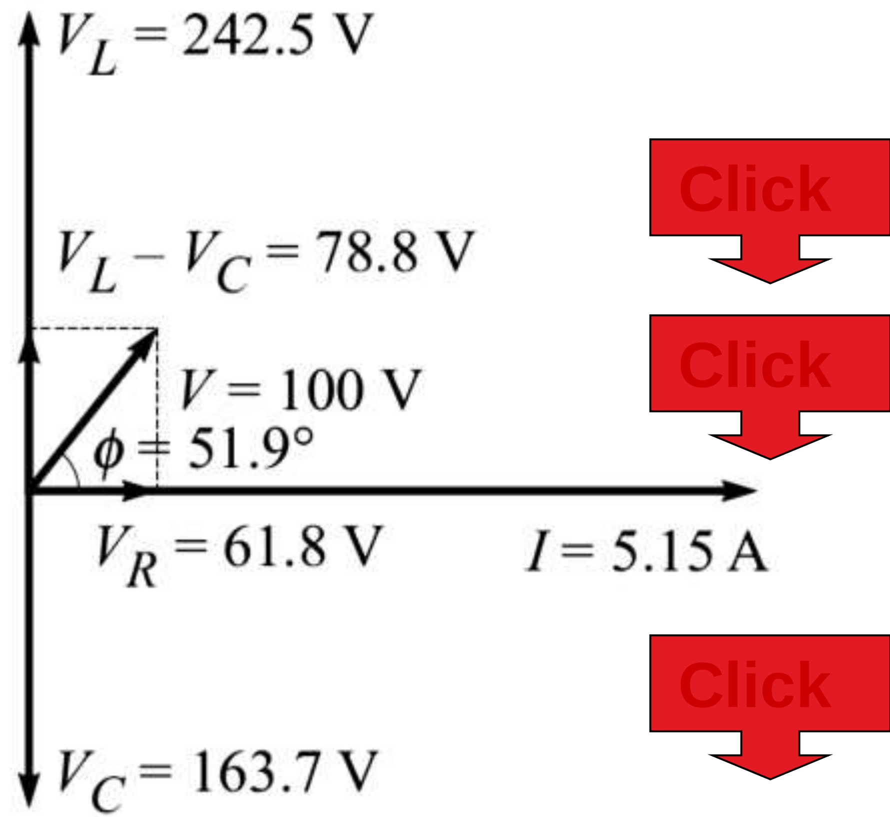
$$pf = \cos 51.9° = \mathbf{0.617\ lagging}$$

(f) The apparent power,

$$P_{app} = VI = 100 \times 5.15 = \mathbf{515\ VA}$$

(g) The average power,

$$P_{avg} = VI\cos 51.9° = \mathbf{317.75\ W}$$



Phasor Diagram

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Example 5

A 100-Ω resistor is connected in series with a choke coil.

When a 440-V, 50-Hz ac voltage is applied to this combination, the voltage across the resistor and the choke coil are 200 V and 300 V, respectively.

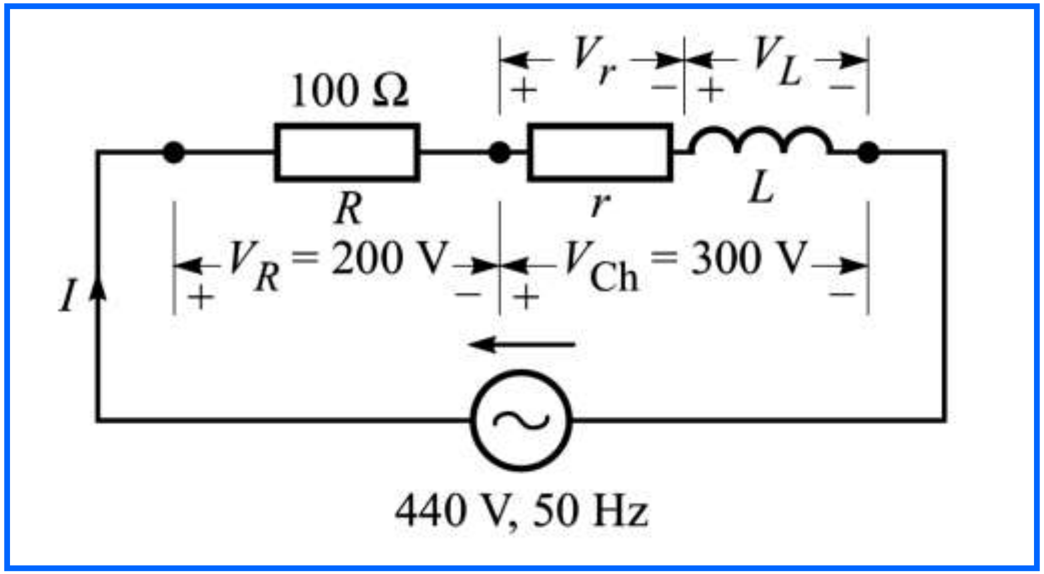
Find the power consumed by the choke coil.

Sketch a neat phasor diagram, indicating the current and all voltages.

Next

Solution :

- A choke coil has some resistance, say *r*, and some inductance, say *L*.
- Because of this resistance *r*, the voltage V_{Ch} leads the current *I* by an angle less than 90° (say, angle θ).
- Hence, $\mathbf{V} = \mathbf{V}_R + \mathbf{V}_{Ch}$.



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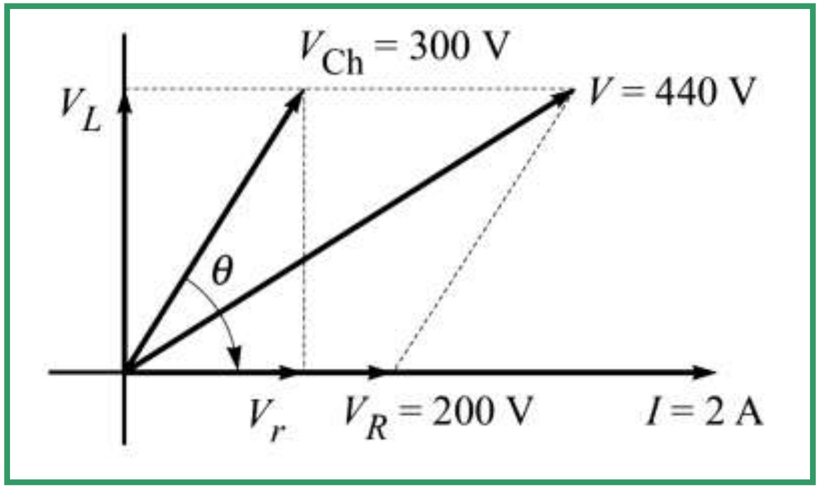

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$$V^2 = V_R^2 + V_{Ch}^2 + 2V_R V_{Ch} \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{(440)^2 - (200)^2 - (300)^2}{2 \times 200 \times 300} = 0.53$$

Click

Now, $I = \dfrac{V_R}{R} = \dfrac{200}{100} = 2\ \text{A};$

Click

$$\therefore\ P = V_{Ch} \times I \times \cos\theta = 300 \times 2 \times 0.53 = \boxed{\mathbf{318\ W}}$$

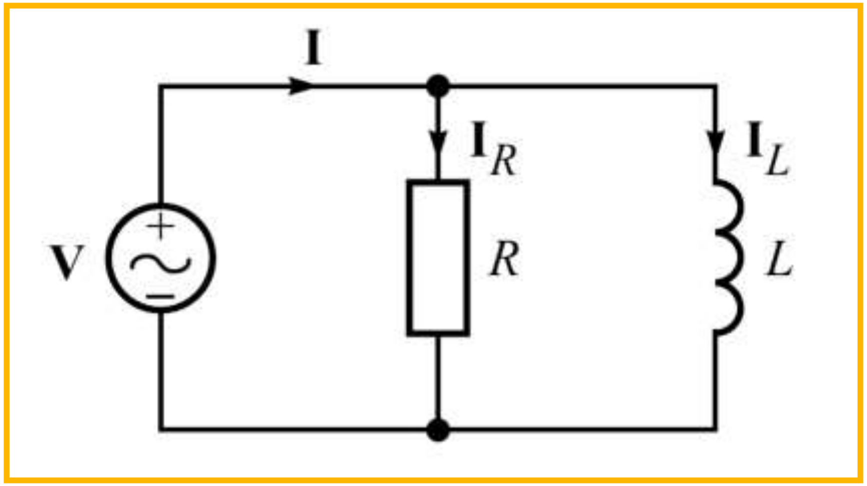
Next

Review

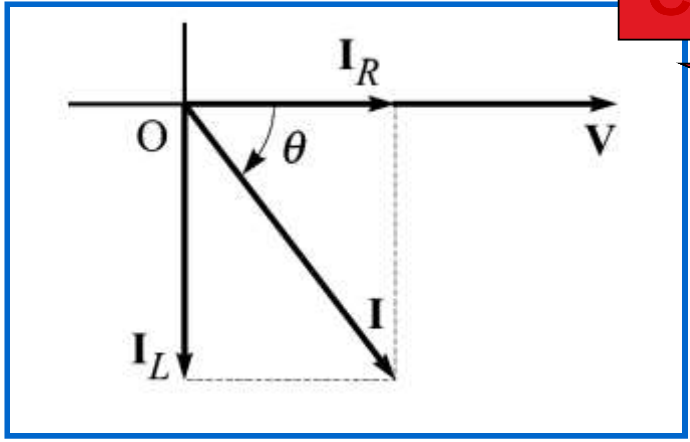
- Series *RL* Circuit.
- How to Draw Phasor Diagram.
- Complex Impedance.
- Impedance Triangle.
- Series *RC* Circuit.
- Complex Power.
- Series *RLC* Circuit.

Out of Syllabus

Parallel *RL* Circuit



(a) The circuit.



(b) Phasor diagram.

Click

$$\mathbf{I}_R = \frac{\mathbf{V}}{R} \quad \text{and} \quad \mathbf{I}_L = \frac{\mathbf{V}}{jX_L} = \frac{\mathbf{V}}{j\omega L}$$

Click

$$\therefore \quad \mathbf{I} = \mathbf{I}_R + \mathbf{I}_L = \frac{\mathbf{V}}{R} + \frac{\mathbf{V}}{j\omega L} = \mathbf{V}\left(\frac{1}{R} - j\frac{1}{\omega L}\right) = \mathbf{VY}$$

Next

Y is the complex admittance of the parallel *RL* circuit,

$$\mathbf{Y} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - j\frac{1}{\omega L}$$

$$\mathbf{Y} = G + jB = (\text{conductance}) + j(\text{susceptance})$$

For parallel *RL* circuit, $G = 1/R$ and $B = -1/\omega L$

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Parallel *RC* Circuit

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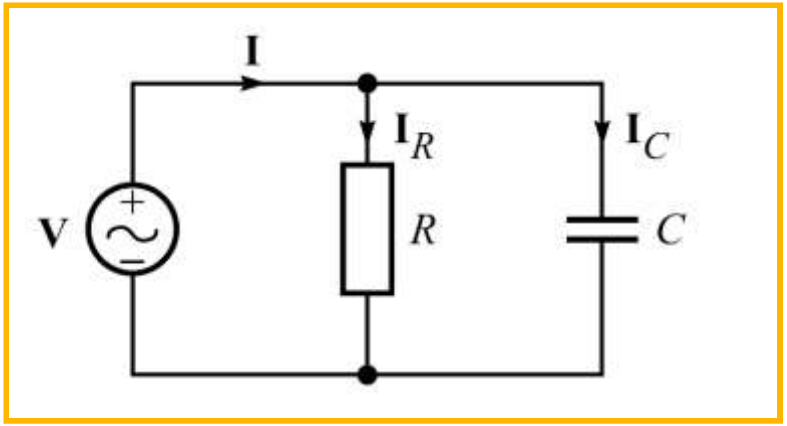
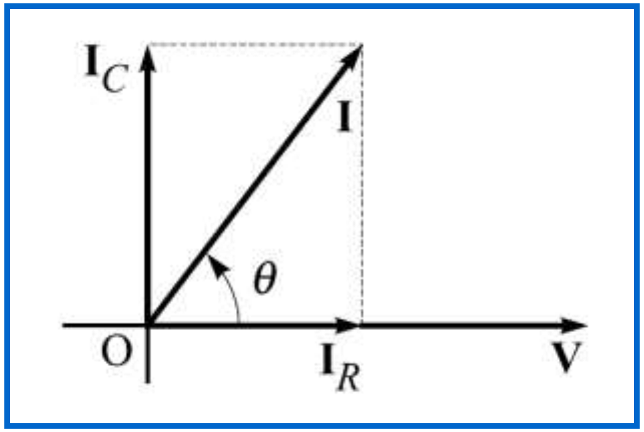


(*a*) The circuit.



(*b*) Phasor diagram.

$$\mathbf{I}_R = \frac{\mathbf{V}}{R} \quad \text{and} \quad \mathbf{I}_C = \frac{\mathbf{V}}{-jX_C} = \frac{\mathbf{V}}{-j(1/\omega C)} = \mathbf{V}(j\omega C)$$

Click

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C = \frac{\mathbf{V}}{R} + \mathbf{V}(j\omega C) = \mathbf{V}\left(\frac{1}{R} + j\omega C\right) = \mathbf{VY}$$

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The complex admittance,

$$\mathbf{Y} = (G + jB) = \left(\frac{1}{R} + j\omega C \right)$$

Click

Next

Example 3

When a two-element parallel circuit is connected across an ac source of frequency 50 Hz, it offers an impedance $Z = (10 - j10)\ \Omega$. Determine the values of the two elements.

Solution : The nature of the impedance indicates that the circuit is capacitive. We find the admittance of the circuit,

Next

$$\mathbf{Y} = (G + jB) = \frac{1}{\mathbf{Z}} = \frac{1}{10 - j10} = (0.05 + j0.05)\ \mathrm{S}$$

Click

$$\therefore \qquad G = 0.05\ \mathrm{S} \ \Rightarrow\ R = \frac{1}{G} = \frac{1}{0.05} = \mathbf{20\ \Omega}$$

$$\text{and} \quad B = 0.05\ \mathrm{S} \ \Rightarrow\ C = \frac{B}{\omega} = \frac{0.05}{2\pi f} = \mathbf{159\ \mu F}$$

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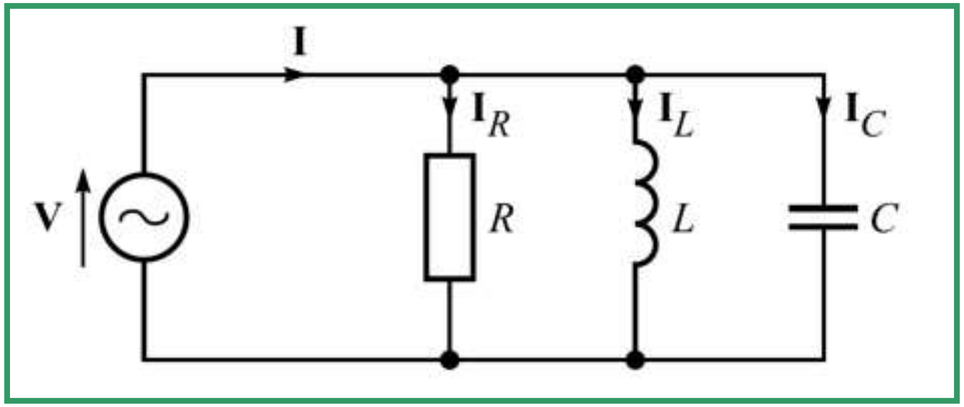
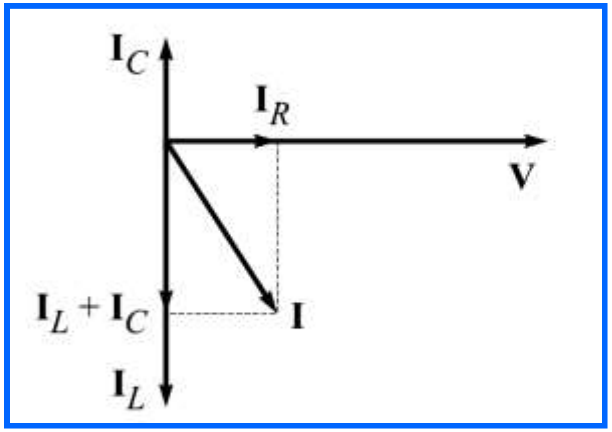
Parallel *RLC* Circuit





(*a*) The circuit.

(*b*) Phasor diagram.

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \frac{\mathbf{V}}{R} + \frac{\mathbf{V}}{jX_L} + \frac{\mathbf{V}}{-jX_C}$$

Click

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$$= \mathbf{V}G + \mathbf{V}(-jY_L) + \mathbf{V}(jY_C)$$

or

$$\mathbf{I} = \mathbf{V}[G + j(Y_C - Y_L)] = \mathbf{V}\mathbf{Y}$$

Obviously,

$$G = \frac{1}{R}; \quad Y_L = \frac{1}{X_L} = \frac{1}{\omega L} \text{ and } Y_C = \frac{1}{X_C} = \frac{\omega C}{1} = \omega C$$

Note that $+j$ is associated with Y_C (and not with X_L) and $-j$ with Y_L.

Next